Estimating Square Roots
A **perfect square** is a number that can be expressed as a product of equal integers.

<table>
<thead>
<tr>
<th>Integer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Square</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Integer</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Square</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
</tr>
</tbody>
</table>
Estimating Square Roots

When we take the **square root** of a perfect square, we obtain the positive integer whose square equals the perfect square.

\[ \sqrt{49} = \]

\[ \sqrt{121} = \]
To estimate the square root of a number that is not a perfect square, we determine the known square roots on either side.

**Example 1** The value of $\sqrt{31}$ is between ___ and ___.
Estimating Square Roots

Example 2  The value of $\sqrt{10} + \sqrt{37}$ is between ____ and ____.
Estimating Square Roots

Note: $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

Example 3  The value of $\sqrt{11} \cdot \sqrt{6}$ is between ____ and ____.
Simplifying Square Roots
To simplify a square root, we look for perfect squares that divide the numerical quantity under the square root.

Example 1  Simplify the radical expression.

\[ \sqrt{200} = \]
Example 2  Simplify the radical expression.

\[ \sqrt{48} = \]
Example 3 Simplify the radical expression.

(a) \( \sqrt{3} \cdot \sqrt{5} = \)

(b) \( \sqrt{18} \cdot \sqrt{72} = \)
Simplifying Square Roots

Example 4  Simplify the radical expression.

\[
\frac{\sqrt{36}}{\sqrt{4}} = \frac{\sqrt{36}}{\sqrt{4}}
\]
Simplifying Square Roots

Example 5  Simplify the radical expression.

(a)  $\sqrt{7} + \sqrt{7} + \sqrt{7}$ = 

(b)  $\sqrt{3} - 8\sqrt{3}$ = 

(c)  $\sqrt{125} + \sqrt{5}$ =
Writing Square Roots in Equivalent Forms
Properties of Square Roots:

1. \( a = \sqrt{a^2} \)

2. \( \sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \)

3. \( \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \)

Illustration:

1. \( 5 = \)

2. \( \sqrt{2} \cdot \sqrt{6} = \)

3. \( \frac{\sqrt{12}}{\sqrt{3}} = \)
Example 1  Which of the following is equivalent to $\frac{\sqrt{32}}{2}$?

(a) $\sqrt{8}$

(b) $\sqrt{14}$

(c) $\sqrt{4}$
Writing Square Roots in Equivalent Forms

Example 2  Which of the following is equivalent to \( \frac{4}{\sqrt{64}} \) ?

(a) \( \frac{1}{\sqrt{16}} \)

(b) \( \frac{1}{\sqrt{4}} \)

(c) \( \frac{2}{16} \)
Writing Square Roots in Equivalent Forms

Example 3  Which of the following is equivalent to \( \frac{\sqrt{432}}{12} \)?

(a) \( \sqrt{420} \)

(b) \( \sqrt{36} \)

(c) \( \sqrt{3} \)
Example 4 Which of the following is equivalent to \( \frac{\sqrt{20} - \sqrt{5}}{\sqrt{20}} \)?

(a) \(-\sqrt{5}\)

(b) \(\frac{1}{\sqrt{5}}\)

(c) \(\frac{1}{2}\)
Example 5  Which of the following is equivalent to the following?

\[
\frac{\sqrt{2 \cdot 10}}{\sqrt{10}} \cdot \frac{\sqrt{10^2}}{\sqrt{4 \cdot 10^2}}
\]

(a) \( \frac{\sqrt{10}}{2} \)

(b) \( \frac{1}{\sqrt{2}} \)

(c) \( 1 \)
Simplifying Square Roots Involving Variables
Example 1  Simplify the radical expression.

\[ \sqrt{r} \sqrt{rz^2} = \]
Simplifying Square Roots Involving Variables

Example 2  Simplify the radical expression.

$$\frac{t}{\sqrt{t^3 r}}$$

Note:  \(a = \sqrt{a^2} = \sqrt{a} \cdot \sqrt{a}\)
Example 3  Simplify the radical expression.

\[ \sqrt{b} \cdot \sqrt{b} + \sqrt{b} \cdot \sqrt{b} \cdot \sqrt{b} \cdot \sqrt{b} = \]
Simplifying Square Roots Involving Variables

**Example 4** Simplify the radical expression.

\[
\frac{\sqrt{2m}}{\sqrt{m}} \cdot \frac{\sqrt{m^2}}{2m^4} = \]

**Note:** \( a = \sqrt{a^2} = \sqrt{a} \cdot \sqrt{a} \)
Example 5  Simplify the radical expression.

\[
\frac{\sqrt{k} + \sqrt{k}}{\sqrt{k + k}}
\]